SINGLE TRIAL PROBABILITY APPLICATIONS: CAN SUBJECTIVITY EVADE FREQUENCY LIMITATIONS?

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Introduction

CROVELLI (2009) ARGUES THAT Richard von Mises was mistaken in defining probability in terms of frequency distributions confined by uniform trials (i.e., collectives). Mises (1957) demonstrates that data historically retrieved from collectives must have a convergent and stable distribution to be used statistically. Hence, this same data can only be applied to collectives, not to individual cases. It follows that the application of probability to a single trial (i.e., a boxing match) cannot be undertaken, even if we do have a series of historically similar boxing matches with which to create a proximate probability distribution. As an alternative, Crovelli posits that probability theorists, "must define probability subjectively; that is, as a measure of our uncertainty about the likelihood of occurrence of some event or phenomenon, based upon evidence which need not derive solely from past frequencies of 'collectives' or classes" (2009, p. 3).

Several ambiguities and confusions in Mr. Crovelli's paper lead to erroneous conclusions regarding the use of frequency distributions in probability theory. This paper shall briefly clear-up these misunderstandings in four steps.

First, Mises's (1957) frequency interpretation of probability will be demonstrated not to be an erroneous definition leading to strict limitations. In fact, Richard von Mises wrote his book, *Probability, Statistics, and Truth*, in order to place probability theory on firmer ground—one based upon a definition consistent with its application. Second, Crovelli's confusion as to

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what uncertainty is (whether ontological i.e., Davidson (1996) or epistemological i.e., Mises (1949)), leads to the conclusion that probability is a tool used to "deal with ... uncertainty" (2009, p. 7). In fact, the distinction between risk and uncertainty seems to evade Crovelli, leading to compounding troubles which require clarification. Third, the causality of events need not lead to the conclusion that any event in a collective need have strictly identical causal factors, and hence, identical outcomes. We will conclude by summarizing the findings, and reinforce the need for probability to be defined and expressed *solely* in terms of frequency distributions.

The necessity of a frequency interpretation for probability theory

Crovelli (2009, p. 6) takes issue that Richard von Mises confines the application of probability to collectives at the expense of "singular events and phenomena."[1] One of Mises's stated goals in his treatise was to define probability in more exact and concrete terms. Indeed, as he explains early on:

> The word "probable" is frequently used in everyday speech. We say, for instance, "It will probably rain tomorrow," or, "It is probably snowing right now in Iceland," or, "The temperature was probably lower a year ago today than it is today." We have no difficulty in explaining what we mean by these statements as long as the inquirer is satisfied by a "descriptive" answer. (1957, p. 2).

While these subjective probability assessments may be preferred by some, they lack any terminological rigor with which to use them fruitfully. Indeed, even today, many probability theorists when asked what probability is will respond by restating how it is calculated.

It becomes apparent that a priori probability distributions cannot be ascertained without verification through empirical tests. Taleb (2007) gives one example of this necessity by asking what an individual would expect to occur if a coin had just been tossed and come up tails several consecutive times. Statistically, the trials are independent. Hence, the a priori probability for the next trial is still 50:50. However, it may become evident that if a coin is tossed repeatedly and continually favors one side over the other that the a

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[1]Although Crovelli classifies both Mises brothers as frequency distribution probability theorists, the literature is less certain. Hoppe (2007) agrees with this assertion, while van den Hauwe (2008) finds disagreement. The best evidence that Ludwig von Mises disagreed with his brother Richard on this issue may be found in his disparaging remarks towards the "neo-indeterminist" school of physics, of which Richard was a part of (see L. von Mises 1957, p. 88). These arguments are, however, outside the scope of the current paper, which wishes only to concern itself with the original question of whether frequency distributions of probability should be replaced with subjective assessments.

priori distribution is incorrect, and that the coin is not fairly weighted; this conclusion can only be determined through empirical trials.[2]

Crovelli (2009, pp. 5–6) takes issue with the fact that although Richard von Mises aimed to develop a theory of probability, he succeeded in merely promoting a *conceptual* definition of probability. This arises due to a limiting factor that Mises imposes regarding historical trials, that of the necessity of an "indefinitely prolonged sequence of observations" (1957, p. 221). If we read beyond the brief summary Mises provides in the final pages of his book which Crovelli cites as proof of this necessity, we find that a further clarification is provided earlier, on pages 83—84, whereby Mises explicitly clarifies this point:

> I must defend myself most emphatically against the recurring misunderstanding that in our theory infinite sequences are always substituted for finite sequences of observations. *This is of course false.* In an example discussed at the end of the preceding lecture, we spoke of the group of twenty-four throws of a pair of dice. Such a group can serve as the subject of our theory, if it is assumed that is has been repeated, as a whole, an infinite number of times and in this way has become an element of a collective. [emphasis added][3]

Indeed, as Mises (1957, pp. 28–29) outlined in his second proposition concerning probability, there are two criteria that the collective must meet.[4] First, the relative frequencies of observations must tend towards a fixed limit. Second, these fixed limits are not affected by the place selection of an outcome (i.e., all events are independent). Neither of these implies that an infinite series of trials be undertaken to establish a stable frequency probability, only that a *sufficient* number be undertaken to establish the fixed limit(s).

Last, Crovelli asserts that probability can be applied to unique trials, and not confined to repeatable series as frequency probability theorists contend. For example, the fact that casinos and bookies seem to successfully profit from applying probabilities to the odds they offer on unique events (i.e., boxing matches) provides visible proof that accurate probability distributions can be calculated for singular events. However, while the

[2]Likewise, Mises (1957, p.71) contends that a die cannot be known to have six equally weighted probabilities unless it has been subject to a sufficiently long series of experiments to demonstrate the fact.

[3]The ideal case would be an infinite series of trials. However, in light of practical limitations this would place on any theory, Mises further clarifies in lecture five the use of finite trials in the calculation of probabilities.

[4]The collective is defined as being "a mass phenomenon *or* an unlimited sequence of observations" (Mises 1957, p. 28, emphasis added)

appearance of odds may seem like a probability estimate of the fight's outcome, this is merely an illusion fabricated by an odds-maker. In distinction, the odds-maker only has to have an estimate of who will win and who will lose a fight.[5] The odds established are used to entice individuals to bet against the expected winner, in hopes of pocketing more winnings. The degree of entrepreneurial forecasting will determine how well the odds-maker has persuaded betters to erroneously choose the wrong player. The odds-maker does not partake in an exercise in probability, but rather one of entrepreneurial forecasting.

To be sure, past matches may be used to aid the entrepreneur in assigning his or her expected likelihood as to which contestant will emerge victorious from a particular boxing match. However, to say that there are 2:1 odds that boxer *A* will out-spar boxer *B* does not imply that the bookie expects boxer *A* to be victorious two-thirds of any one fight, nor that the final outcome will be two-thirds of a victory for *A*, and one-third for *B*. For unique events, there can only be one outcome which may not rely on the established frequency distribution.[6] Instead, the odds are entrepreneurially forecast based upon an expectation that boxer *A* will prevail, *and* that the greatest profit will be earned by enticing individuals to seek higher pecuniary rewards by betting for the expected loser in the fight.

Risk and uncertainty: rehashing some old themes

Crovelli (2009, pp. 7–8) includes a whole sub-section entitled "Uncertainty" yet it remains unclear if he grasps the real definition the chosen heading. In the section's opening paragraphs, he claims "[i]t is vital to note that when we deal with the subject of probability we must necessarily and

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[5]Additionally, gambling odds do not reflect bookies' beliefs concerning a likely winner so much as they represent bettors' beliefs concerning the betting patterns of others.

[6]Taleb (2004: 183) relays a similar point, demonstrating that expected probabilities share little in common with the objective reality which will obtain:

> Consider a bet you make with a colleague for the amount of $1,999, which, in your opinion is exactly fair. Tomorrow night you will have zero or $2,000 in your pocket, each with a 50% probability. In purely mathematical terms, the fair value of a bet is the linear combination of the states, here called the mathematical expectation, i.e., the probabilities of each payoff multiplied by dollar values at stack (50% multiplied by 0 and 50% multiplied by $2,000 = $1,000). Can you imagine (that is visualize, not compute mathematically) the value being $1,000? We can conjure up one and only one state at a given time, i.e., either 0 or $2,000. Left to our own devices, we are likely to bet in an irrational way, as one of the states would dominate the picture—the fear of ending with nothing or the excitement of an extra $2,000.

concomitantly deal with the subject of uncertainty. The term "probable" applies to statements and facts about which we are uncertain—the word does not apply to statements and facts about which we are absolutely certain" (p. 7). That there are only two options available—complete-certainty and complete uncertainty—seems to neglect the case of risk.[7]

Frank Knight (whom the author does not cite) delineates the distinction between risk and uncertainty in terms of the ability to quantify outcomes (1921, p. 21). Hence, risky are those multiple items in a set which we know to exist, and may measure accordingly. Uncertain outcomes, in distinction, are fundamentally unknown, and hence uncertain.[8] This distinction appears to not be considered by Mr. Crovelli as he states: "It is precisely because man is uncertain about many past and future events, outcomes, statements and phenomena that the need for probability arises" (p. 7). Indeed, this statement takes on diminished importance in light of Knight (1921, p. 225), as he so succinctly phrases the issue: "The distinction between risk and uncertainty is not because there is no basis for assigning probabilities, but as there is no valid basis of any kind for classifying instances."

Uncertainty represents those elements that we do not know that we do not know exist. There is no way that we may make *any* statement about the outcomes of situations involving complete uncertainty. Risk, in distinction, represents those outcomes which we know to exist, and belong to a class in which alternative outcomes may obtain. If these are measurable and repeatable, we may apply probability theory to them to ascertain a statement as to which we consider will prevail over a set of trials.

Mr. Crovelli relies heavily on uncertainty and quotes primarily from two authors whose views on the topic, as will be shown, are ambiguous in the first case, and erroneous in the latter.

First, Ludwig von Mises is quoted concerning what necessitates a probable statement—a limitation of knowledge (1949, p. 107). On this same page, Mises forwards two drastically different concepts. In the first instance, he categorizes "class probabilities" as those which we assume to know everything about the behavior of a class of outcomes (i.e., Richard von Mises "collectives"). Several pages later, Mises introduces the category of "case probabilities" (p. 110). These are those events which we know some of the

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[7]It is also a naïve and fruitless assumption, a point Garrison (1982, p. 132) elucidates.

[8]The discussion at hand need not concern itself with whether uncertainty is fundamentally ontological in nature (i.e., Keynes 1921, Davidson 1991; 1996) or epistemological (i.e., Mises 1949; O'Driscoll and Rizzo 1985). Langlois and Cosgel (1993) argue that Knight (1921) held both views.

factors (i.e., partial knowledge) regarding an outcome, but there are other determining factors of which we know nothing. As he (1949, pp. 110–13) clarifies the difference:

> Case probability has nothing in common with class probability but the incompleteness of our knowledge. In every other regard the two are entirely different... Everything that outside the field of class probability is commonly implied in the term probability refers to the peculiar mode of reasoning involved in dealing with historical uniqueness or individuality, the specific understanding of the historical sciences... Case probability is not open to any kind of numerical evaluation.

Despite the unfortunate use of a similar name to signify two distinct concepts (i.e., case and class probability) and the fact that the concept of probability is not applicable to one of these concepts (i.e., case probability), Mises is quite clear on the implications of the distinction.

Later, Crovelli (2009, p. 8) quotes Anthony O'Hagan for his views on probability. However, O'Hagan's views are deficient as they make no meaningful distinction between risk and uncertainty. Indeed, he makes his views toward uncertainty clear stating: "Accurate weighing of uncertainty is vital to good decision-making at every level" (1988, p. 2). The problem, which Knight, Keynes, and the brothers Mises, have previously made clear is that there is no method of weighing uncertainties, they are, after all, things about which we, to borrow Keynes's famous words, "just don't know."[9] O'Hagan assumes uncertainty arises from different "states" of the future world which are known, even if their exact probabilities are not, while uncertainty-theorists realize that there is no way to either: a) know, *or* b) categorize, these future states.

Frequency probability theorists see the distinction between risk and uncertainty as key to understanding class probability (applicable to the natural world) and case probability (applicable to decision-making). By contrast, the subjectivist view of probability does not see the risk-uncertainty distinction as relevant for probability theory—the distinction is not between two types of probability, but rather between two kinds of information structures (Langlois 1982, p. 6). This lack of distinction is problematic as it fails to recognize that uncertainty and knowledge are exclusive (Shackle 1961, p. 60). Indeed,

[9]This bifurcation is not unique to O'Hagan; almost every probability text makes the same confusion. Hirshleifer and Riley (1995, p. 10) open their text by stating: "[W]e disregard Knight's distinction [between risk and uncertainty], which has proven to be a sterile one. For our purposes risk and uncertainty mean the same thing." The reason is not "sterile" but rather lies in the difficulties inherent in modeling those things we lack knowledge of—uncertainties.

uncertainty may be referred to as *un*knowledge—we do not know something to exist, and we have no knowledge that we lack knowledge of its existence. To treat uncertainty probabilistically as if it were a similar knowledge deficiency as are cases of risk overlooks the fact that it is not a partial deficiency, but rather an absolute one that we are not even aware of.

Collectives and causation

One of Carl Menger's greatest contributions to the social sciences occurred in the first line of his Principles treatise: "All things are subject to the law of cause and effect. This great principle knows no exception" (1976, p. 51). Crovelli (2009, pp. 11–15) applies this principle to the frequency theory of probability in an attempt to show that the method is flawed. Concerning Richard von Mises's definition of the collective necessary for frequency tabulation, he claims:

> This criterion for the employment of the relative frequency method has two important implications: 1) the events must be virtually identical with one another, such that they can conceptually be categorized together as members of a "collective," and 2) the events must not be exactly identical to one another in every conceivable way, or else there would be no variation in outcome. (2009, p. 13)

Indeed, if a collective was defined as having individual items being identical in all ways, Mr. Crovelli would be quite correct that the probability of all outcomes would be the same and equal to unity. Fortunately for frequency probability theorists, Richard von Mises (1957, p. 12) already clarified this point, stating:

> [The collective] denotes a sequence of uniform events or processes which differ by certain observable attributes, say colours, numbers, or anything else... All the throws of dice made in the course of a game form a collective wherein the attribute of the single event is the number of points thrown... The definition of probability which we shall give is only concerned with "the probability of encountering a certain attribute of a given collective."[10]

If we revisit for a moment the concept of risk, we may remember that it is defined by a knowledge deficiency whereby, according to Ludwig von Mises (1949, p. 107), "[w]e know or assume to know … everything about the

[10]Hoppe (2007, p. 19) follows this reasoning, arguing that for items such as dice and bottles, we know all possible attributes, and hence are able to group them into a collective. Once we have attained a "closed" collective, it becomes possible to give operational meaning to Richard von Mises's notion that a "sufficiently long" series of events is able to obtain a stable outcome limit.

behavior of a whole class of events or phenomena; but about the actual singular events or phenomena we know nothing but that they are elements of this class." Hence, we know that attributes are similar to allow an event to be included in a class, and also we may know *some* of the causal factors that influence the outcome. However, if we had full knowledge of the causal factors, we would be certain, as Mr. Crovelli correctly points out, of the final outcome—we would not have a probable statement but a certain one.

As Richard von Mises (1957, pp. 23–25) demonstrates, the element of independence and randomness between the factors affecting outcomes is what necessitates the probabilistic approach. Hence, as he (p. 24) clarifies: "an appropriate definition of randomness can be found without much difficulty. The essential difference between the sequence of the results obtained by casting dice and the regular sequence of large and small milestones consists in the possibility of devising a method of *selecting the elements* so as to produce a fundamental change in the relative frequencies."

The existence of time-invariant causal laws does not refute the arguments frequency theorists base their theory of probability on. Instead, as has been explained, it is our incomplete knowledge of the *individual* items in a collective in light of our more complete knowledge of the greater collective that allows us to use the empirically established outcome limits to deduce future outcomes.

Probability—What Is It?

Although it may seem strange to define our central subject matter this late in the paper, it now proves instrumental to tie the previous arguments together into a coherent definition of probability. The distinction between risk and uncertainty becomes instrumental. Uncertainty is not just an extreme degree of risk, as many subjectivist probability theorists allude to. Instead it represents a fundamentally different category, whereby it is not that our knowledge of an event or object is deficient; rather, it is that we lack knowledge of the deficiency.

Entrepreneurs have a method of dealing with uncertainty, first delineated by Knight (1921). Accurate entrepreneurial forecasting under these conditions results in entrepreneurial profits. This cannot be, however, undertaken probabilistically as these uncertain events rule out any knowledge we may have of the outcomes. Any application of probability will require that a partial knowledge be known concerning an event, forcing the application to apply to risky scenarios only.

In contrast, risk arises from a deficiency in knowledge resulting in events of which we have sufficient knowledge. We may group these events into classes, where we lack some knowledge concerning the individual events within the class. Within this class we have a tool—probability theory—which allows us to forecast results from future trials of repeated events. This arises as the class is comprised of more or less homogenous events which will tend to a stable outcome limit. However, to make use of the collective which we know have partial knowledge of, two caveats must be abided by.

First, the stable limit(s) of the collective must be identified. In order for this limit to be reached, prolonged series of trials must be undertaken which approach this limit. Events which cannot be repeated so as to test this limit cannot be considered as part of the collective, as they cannot contribute to the establishment of the probabilistic properties of it—namely, the limit(s) and number of events contained in the collective.

Second, as a corollary, a subjectivist approach to probability interpretation cannot apply to these collectives. As the subjective approach relies on establishing or using a probability distribution on unique events, they are excluded from the collective by definition. Hence, only a frequency approach to probability may be applied to risky events which we have incomplete knowledge about.

Using a subjective interpretation of probability loses the generality of the frequentist approach, while offering no significant advantages in replacement. Indeed, as Klein (2009: 33) explains: "It is not clear exactly what is gained by redefining probabilities as 'subjective with one information set' or 'subjective with another information set.'" Once we adopt a definition of probability based on subjective knowledge sets, the definition loses all operational meaning, as it implies nothing other than individual entrepreneurial forecasting—something more aptly suited to describing decision-making under uncertainty.

Concluding remarks

Much of the Mr. Crovelli's arguments are summarized in his concluding statements:

> I have argued that the definition of probability depends upon whether the world is governed by time-invariant causal laws, or whether there exists in the world an element of uncaused randomness. I have argued that if the world is governed by time-invariant causal laws, then we must ascribe any uncertainty man might have about those causes to human ignorance alone and not to some property of the world, and that we would be consequently obliged to define probability as a subjective measure of man's beliefs about the causal factors at work in the world. (p. 15)

Indeed, we find much agreement with Mr. Crovelli's first statement, but decidedly less with his second. The world is governed by causal forces, and as a result, we may ascribe uncertainty to our limited knowledge of these laws, or their embodied factors. However, from this it does not follow that we are "obliged" to define probability as a subjective measure of our beliefs concerning as yet unrealized outcomes. Indeed, Mr. Crovelli himself must realize this, as he continues to states that the objective frequency *and* subjective probability theorists may coexist peacefully. Well, we may ask, which one is it? The arguments frequency probability theorists use negate any co-existence with the subjective approach. Any resolution must adopt one or the other.

When we realize the distinction between two similar concepts—risk and uncertainty for Frank Knight, case and class probabilities for Ludwig von Mises, and collectives and unique events for Richard von Mises—we understand that probability is not something which may be redefined, as Crovelli (p. 15) assumes. Richard von Mises sought to explain what is different about the possible treatments of collectives and unique events concerning their probability characteristics while making his original justification for the frequency approach.

The subjective probability approach that Mr. Crovelli has forwarded finds more agreement with the "possibility" approach that G. L. S. Shackle (1952) developed to deal with unique events, than with probability theory proper. Indeed, for these unique events that fail to belong to a collective, Shackle's alternative approach evades all the aforementioned deficiencies of Mr. Crovelli's subjective approach. For brevity's sake, we refer the interested reader to these arguments.[11]

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[11]First developed in Shackle (1952), further refinements and expositions can be found in Shackle (1958) and (1961).

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